Image of n-dimensional ball by any linear mapping.

Hello,

Let $f : \mathbb{R}^n \to \mathbb{R}^m$ a linear map and let $B \subset \mathbb{R}^n$ be the ball of radius 1 centered at the origin. So, is f (B) an ellipsoid? If so, how can the half-axes be determined?

Thank you in advance.

(forum : les-mathematiques.net)

Answer :

It is necessary to geometrize what really happens instead of opening the large suitcases.

f(B) restriction to the supplementary of Ker (f) is not an ellipsoid in the same way a hatched disc is not a disc.

We consider in the general case, (A) the n-cube circumscribed to B.

-The image of (A) on the supplementary of Ker (f) is the image of a k-cube on this restriction by an automorphism of R ^ k, (k = rg (f), even if you draw the most distorted object you want it doesn't matter,it will be preserved)

What interests us is that on the kernel of f, any vector of this space is reduced to 0, we reformulate:

-We construct (A): first we start with a k-cube (antecedent by an automorphism of R ^ k of f (A) (f ^ -1 (f (A) on the restriction)), now let's construct the true antecedent of A by f, then f (B) :

It is as if we "hatch" of (A) Ker (f),it is the image of this new figure (A$) by the automorphism of R ^ k defined above and therefore f (B) is an ellipsoid of dimension k with at most (n-k) points contracted at 0.

From a distance it is like a k-ellipsoid but in the neighborhood of its kernel its image degenerates monotonously towards the origin.